Algebra One – Chapter 7
Word Problems – Textbook pp382-383 #25, 27, 28, p388 #23, 48

Problem 23 p382

A seafood restaurant owner orders perch and salmon. He wants to buy at least 50 pounds of fish, but he cannot spend more than $180. Write and graph a system of inequalities to show the possible combinations of perch and salmon he could buy. (Note: Perch costs $4/lb, salmon $3/lb)

Solution:
We're trying to figure out ***how much*** (in pounds) of each type of fish we might buy.
Variables: P = number of pounds of perch
S = number of pounds of salmon

So we will prepare a graph where P and S are the two axes. Let's put **P along the horizontal** axis and **S on the vertical axis**. (We could have chosen the other way; it doesn't matter which way we orient our axes, so long as we are consistent.)

Note that on the P-S graph we won't be visiting any points with negative values. Our graph will include only the first quadrant.

Limitations: Money: (Money spent on perch) + (Money spent on salmon) ≤ (Money available)

$$4P + 3S \le 180$$

Temporarily we will change this to an equality: $4P + 3S = 180$
Where does this equation cross the P-axis and where does it cross the S-axis?
If we spend all our money on perch, we'll get 45 pounds (and no salmon).
On our P-S graph, this is the point (45,0)
If we spend all our money on salmon, we'll get 60 pounds (and no perch).
On our P-S graph this is the point (0,60).
Draw a ***solid*** line between these two points (solid because our limit is less than or equal to). This solid line will make up part of our boundary.

Go back to the inequality: On which side of the money-limit boundary must we be?
We can't spend *more* than $180, but we might choose to spend *less*.
What if we buy only 55 pounds of salmon and no perch. Does this break our budget?
No. So we want the arrows on our money limit to point in the direction of that point, below the money-limit boundary.

Quantity: (Pounds of perch) + (pounds of salmon) ≥ (fifty pounds)

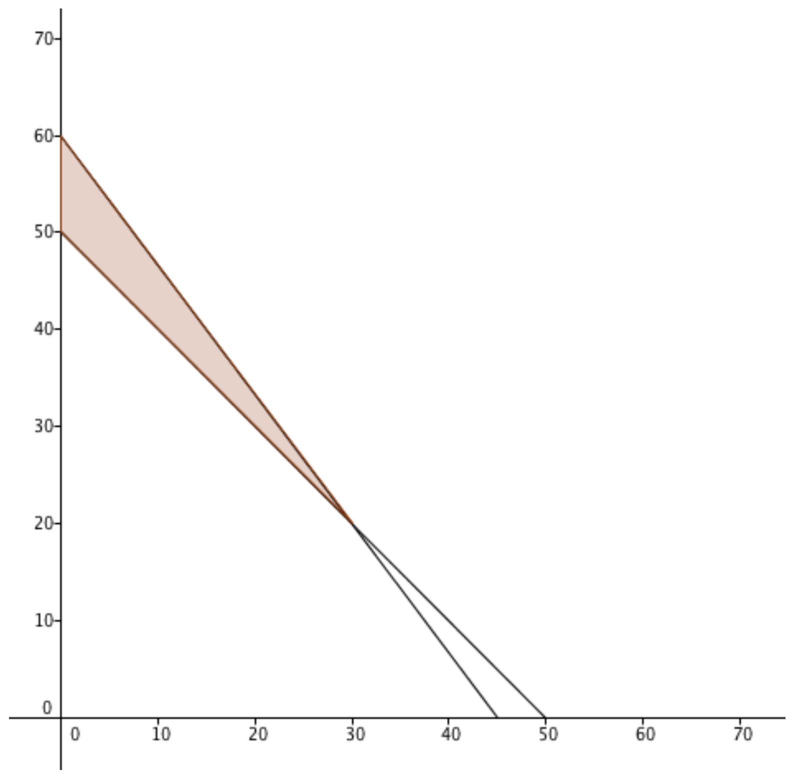
$$P + S \ge 50$$

Temporarily change this inequality into an equation: $P + S = 50$
This line crosses the P axis at 50, so put a point at (50,0).
This line crosses the S axis at 50, too, so put a point at (0,50).
Draw a ***solid*** line between these two crossing points. This line will form another part of the boundary.
We need to be **above** this line to ensure we have enough fish for dinner

Problem 23 p382, continued

Overlap region
There is a skinny, triangular region that satisfies both constraints. One corner lies at the (30,30) P-S point, another at (0,50), and another at (0, 60). All these points are solutions, as are the points in between the corners and points inside this triangular region, such as (6,45) or (15, 40).

Here is a simple graph of the constraints and the solution to this problem. Our restaurant owner will be buying ***at least*** 30 pounds of salmon, maybe more. The ***most*** perch he can buy is 30 pounds.



Problem 48 p383

A drum maker sells two sizes of drums. A 14-inch drum sells for \$180 and an 18-inch drum sells for \$240. He is trying to decide how many drums to build and considers the following:

- He wants to produce and sell at least \$2700 worth of drums.
- He has materials to make no more than 17 drums.
- He plans to make more 14-inch drums than 18-inch drums.
- He wants to make at least four 18-inch drums.

a. Write and graph the four inequalities that describe this situation.
b. Give one possible solution to the system.

Solution

Variables: B = the number of big drums that might be made
S = the number of small drums that might be made

So we will prepare a graph where B and S are the two axes. Let's put **B along the horizontal** axis and **S on the vertical axis**. (We could have chosen the other way; it doesn't matter which way we orient our axes, so long as we are consistent.)

Note that on the B-S graph we won't be visiting any points with negative values. Our graph will include only the first quadrant.

Limitations:

Money: (Money made on big drums) + (Money made on small drums) ≤ (Money available)

$$240\text{ B} + 180\text{ S} \geq 2700$$

Temporarily we will change this to an equality: 240B + 180S = 2700.

Now, if only small drums are made, then the maker will have to sell 2700/180, or 15.
If only big drums are made, then the maker will have to sell 2700/240, or 11.25 of them.
(Yes, it's pretty hard to sell one-quarter of a drum, but for purposes of plotting the line the B-axis intercept lies at the (B, S) point of (11.25,0).}
We want to be on the upper right side of this line if we're to make ***more than*** \$2700.

Materials: At most the maker can produce 17 drums (of whichever kind).

$$\text{B} + \text{S} \leq 17$$

Temporarily change this to an equality and plot the line: B + S = 17.
Plot points at (17,0) and (0,17) as the two extremes, then draw a solid line between.
The maker can choose to make fewer drums, but he cannot make more. We want the region to the lower left of this boundary.

Limitations, cont'd

Drum choice: The maker wants to make more small drums than large drums:

$$S > B$$

Temporarily change this to an equality and plot the line: S = B. This is a line that starts at (0,0) and goes up at a 45 degree angle, just like the line y = x. Since this constraint is a strict inequality (no "or equal to" part), the boundary line must be **dotted**. And since S should be larger than B, we will want the region above and to the left of this dotted line.

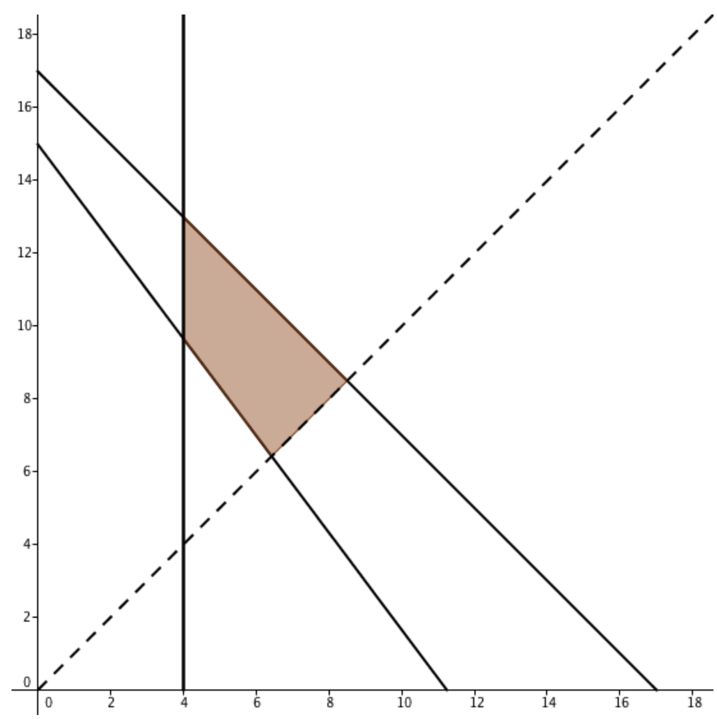
Big drum requirement: The maker wants to make at least four big drums:

$$B \geq 4.$$

Temporarily change this to an equality and plot the line: B = 4.
Since B is plotted on the horizontal axis, this line will be vertical, rising from (4,0). Also, it will be a solid line. Since B must be larger than or equal to 4, select the region to the right of this line.

Overlap region
There is a blocky, irregular region that satisfies all four constraints. Only one border is dotted; the others are solid.

Here is a simple graph of the constraints and the solution to this problem.



One possible solution in this shaded region is (6, 10): six large drums and ten small ones.

Problem 25 p388

A furniture finish consists of turpentine and linseed oil. It contains twice as much turpentine as linseed oil If you plan to make 16 fluid ounces of furniture finish, how much turpentine do you need?

Solution
Let T be the amount of turpentine used and let L be the amount of linseed oil used.

$T + L = 16$
$T = 2L$

$(2L) + L = 16 \rightarrow L = 5.33 \rightarrow T = 10.66$

Problem 27 p388

The perimeter of a rectangle is 114 feet. Its length is three feet more than twice its width. Find the dimensions of the rectangle.

Solution:
Let L be the length and W be the width.

$2L + 2W = 114 = P$
$L = 2W + 3$

Use the second equation to substitute for L in the first equation:

$2(2W + 3) + 2W = 114 \rightarrow 4W + 6 + 2W = 114 \rightarrow 6W = 108 \rightarrow W = 18$ feet.

Back substituting 18 for W in the second equation reveals L = 39 feet .

Check with the first equation: $2(39) + 2(18) = 114$? $\rightarrow 78 + 36 = 114$? $\rightarrow 114 = 114$. Okay!

So the rectangle is 39 feet long and 18 feet wide.

Problem 28 p388

Marcella and Rupert bought some party supplies. Marcella bought 3 packages of balloons and 4 packages of favors for \$14.63. Rupert bought 2 packages of balloons and 5 packages of favors for \$16.03. Find the price of a package of balloons.

Solution:
We are looking for the prices of the two types of items.
Let B be the price of a package of balloons.
Let F be the price of a package of favors.

$3B + 4F = 14.63$
$2B + 5F = 16.03$

Use elimination. Multiply the top equation by 2 and the bottom equation by 3:

$6B + 8F = 29.26$
$6B + 15F = 48.09$

Subtracting the bottom from the top gives $-7F = -18.83 \rightarrow F = 2.69$.
Substituting back into the top equation:
$3B + 4(2.69) = 14.63 \rightarrow 3B = 3.87 \rightarrow B = 1.29$

Checking the second equation: $2(1.29) + 5(2.69) = 16.03$? $\rightarrow$ $2.58 + 13.45 = 16.03$ okay!

So a package of balloons costs \$1.29 and a package of favors costs \$2.69.