

Algebra Section 9-5 Notes

Factoring $x^2 + bx + c$

Name: _____

Date: _____ Block: _____

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Suppose you have a polynomial $x^2 + bx + c$.

To understand how to **factor** such a polynomial, look at the following product that has been simplified using **FOIL**:

F O I L

$$\text{Ex: } (x + 2)(x + 5) = x^2 + 2x + 5x + 2 \cdot 5 = x^2 + 7x + 10$$

In the final expression do you see where the 7 and the 10 come from?

The 7 comes from the **addition** of 2 and 5, while the 10 comes from the **product** of 2 and 5.

To break down $x^2 + bx + c$ into its factors, we will need a pair of binomials. Let's call them $(x+p)$ and $(x+q)$.

These two binomials will have the following characteristics:

1. The **product** of p and q must equal c .
2. The **sum** of p and q must equal b .

Note a few things:

1. If c is positive and b is positive, then the two factors of c are positive.
2. If c is positive and b is negative, then the two factors of c are negative.
3. If c is negative, then one of its factors is positive and the other is negative.

Example One: c is positive

Factor $x^2 + 7x + 12$.

Make a small table to explore possible values for our binomials' p and q :

$p * q$	$p + q$
$1 * 12 = 12$	$1 + 12 = 13$
$-1 * -12 = 12$	$-1 + -12 = -13$
$2 * 6 = 12$	$2 + 6 = 8$
$-2 * -6 = 12$	$-2 + -6 = -8$
$3 * 4 = 12$	$3 + 4 = 7$
$-3 * -4 = 12$	$-3 + -4 = -7$

We want the combination (3,4), since that is the only pair whose product is 12 **AND** whose sum is +7:

Check:

$$(x + 3)(x + 4) = x^2 + 4x + 3x + 3 \cdot 4 = x^2 + 7x + 12$$

You try!

1a. Factor $g^2 + 9g + 20$.

1b. Factor $v^2 + 21v + 20$

1c. Factor $a^2 + 13a + 30$

Example Two: Factoring $x^2 - bx + c$

Factor $d^2 - 17d + 42$.

Since the middle term is negative, find negative factors of 42. Identify the pair that has a sum of -17 .

Factors of 42	Sum of Factors
-1 and -42	-43
-2 and -21	-23
-3 and -14	-17
-6 and -7	-13

The third pair seems to work: $(d - 3)(d - 14) = d^2 - 14d - 3d + (-3)(-14) = d^2 - 17d + 42$

You try!

2a. $k^2 - 10k + 25$

2b. $x^2 - 11x + 18$

2c. $q^2 - 15q + 36$

Example Three: Factoring Trinomials With a Negative c

Factor $m^2 + 6m - 27$.

Identify a pair of factors of -27 that has a sum of $+6$:

Factors of -27	Sum of Factors

3a. $m^2 + 8m - 20$

3b. $p^2 - 3p - 40$

3c. $y^2 - y - 56$

Homework: Practice 9-5, #6, 12, 18, 24, ..., 84 (every sixth problem, 14 problems total)